

1.2 Polynomials

Question Paper

Course	CIEA Level Maths
Section	1. Algebra & Functions
Topic	1.2 Polynomials
Difficulty	Hard

Time allowed: 100
Score: /79
Percentage: /100

Question 1a

(a) Expand and simplify $(2 - x)(3x + 1)(x + 1)$.

[1 mark]

Question 1b

(b) A square has side lengths of $(5x - 2y + 3)$ units. Find an expression for the length of the diagonal of the square in terms of x and y .

[2 marks]

Question 2

Given that $(2x - 3y)^2(y - 2x) = ax^3 + bx^2y + cxy^2 + dy^3$, where a, b, c and d are constants, find the values of a, b, c and d .

[2 marks]

Question 3

Factorise completely $15x^3 + 19x^2 - 10x$.

[2 marks]

Question 4

Divide $x^3 - 19x - 30$ by $(x - 5)$.

[2 marks]

Question 5a

$$f(x) = x^3 - 28x + 48$$

(a) Find the remainder when $f(x)$ is divided by $(x - 3)$.

[2 marks]

Question 5b

(b) Given that $(x + 6)$ is a factor of $f(x)$, factorise $f(x)$ completely.

[4 marks]

Question 6a

$$f(x) = 6x^3 - 19x^2 + 11x + 6$$

(a) Show that $f(x) = (2x - 3)(ax^2 + bx + c)$ where a , b and c are constants to be found.

[2 marks]

Question 6b

(b) Hence factorise $f(x)$ completely.

[4 marks]

Question 6c

(c) Write down all the real roots of the equation $f(x) = 0$.

[2 marks]

Question 7a

$$f(x) = 4x^3 - 7x - 3$$

(a) Use the factor theorem to show that $(2x + 1)$ is a factor of $f(x)$.

[2 marks]

Question 7b

(b) Factorise $f(x)$ completely.

[4 marks]

Question 7c

(c) Write down all the real roots of the equation $f(x) = 0$.

[2 marks]

Question 8a

$f(x) = x^3 + rx^2 + sx - 30$. Given that $f(2) = 0$ and $f(-3) = -240$:

(a) find the values of r and s .

[6 marks]

Question 8b

(b) Factorise $f(x)$ completely.

[3 marks]

Question 9a

The function $f(x)$ is given by

$$f(x) = 4x^3 - 7x^2 - 21x + 18$$

(a) Show that $(4x - 3)$ is a factor of $f(x)$.

[2 marks]

Question 9b

(b) Hence, or otherwise, fully factorise $f(x)$.

[4 marks]

Question 9c

(c) Write down the roots of $f(x)$.

[2 marks]

Question 10

Show that $(5x - 2)$ is a factor of $25x^3 + 55x^2 - 56x + 12$.

Hence find all the real solutions to the equation $25x^3 + 55x^2 - 56x + 12 = 0$.

[5 marks]

Question 11a

(a) Given that $(4x - 5)$ is a factor of $4x^3 - 9x^2 + ax + 30$ find the value of a .

[2 marks]

Question 11b

(b) Hence, or otherwise, fully factorise $4x^3 - 9x^2 + ax + 30$.

[2 marks]

Question 12

(i) Find the remainder when $x^3 - 2x^2 + 4x - 3$ is divided by $x - 2$.

(ii) Find the value of $f(2)$ when $f(x) = x^3 - 2x^2 + 4x - 3$.

(iii) Comment on your answers to parts (i) and (ii).

[4 marks]

Question 13a

It is given that

$$\frac{f(x)}{g(x)} = 2x + 3 - \frac{4}{x + 1}$$

(a) Why would assuming that $g(x) = x + 1$ be a logical first step in attempting to determine the precise forms of $f(x)$ and $g(x)$?

[1 mark]

Question 13b

(b) By first making the assumption from part (a), find $f(x)$.

[2 marks]

Question 13c

(c) Explain, with an example, why the forms of $f(x)$ and $g(x)$ determined in parts (a) and (b) are not the only possible forms for those functions.

[2 marks]

Question 14

When $x^3 + ax^2 + 4x - 1$ is divided by $x + 2$ the quotient is $x^2 - 4x + 12$ and the remainder is b .

Find the values of a and b .

[3 marks]

Question 15

Given that $(x + 4)$ is a factor of the function $f(x) = px^3 + (5p + 1)x^2 + 5qx - 2q - 2$ and that the remainder when $f(x)$ is divided by $(x + 1)$ is -12 , find the values of the constants p and q .

[6 marks]

Question 16

Show that $3x^3 + 16x^2 - 22x$ can be written in the form $(3x + 1)(ax^2 + bx + c) + d$, where a , b , c and d are constants to be found.

[4 marks]